RAND

Concept-Level Analytical Procedures for Loading Nonprocessing Communication Satellites with Direct-Sequence, Spread-Spectrum Signals

Edward Bedrosian, Gaylord Huth

Project AIR FORCE
Arroyo Research Division

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United States Army

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PREFACE

The command and control of modern military forces is becoming increasingly dependent on space assets for a wide variety of functions. Prominent among these are communication satellites, the value of which has been amply demonstrated in recent military operations, notably Operations Desert Shield/Storm. Unfortunately, modern communication satellite systems are very expensive. Given the shrinking military budget and the volatile geopolitical world in which they must be used, it becomes essential to obtain those systems that best serve these uncertain needs and to do so at the least cost.

As part of its research for the Army and the Air Force, RAND is constructing a concept-level modeling tool that is intended to permit evaluating conceptual military communication satellite systems at a systems level. That is, it considers only basic design parameters. This report is the second in a series devoted to presenting the analytical procedures required in such a computer model and does not discuss the model's implementation. The first in the series is MR-639-AF/A, *Concept-Level Analytical Procedures for Loading Nonprocessing Communication Satellites with Nonantijam Signals*, by Edward Bedrosian and Gaylord K. Huth, 1996.

This analysis has been conducted jointly under two of RAND's federally funded research and development centers (FFRDCs)—Project AIR FORCE and the Arroyo Center.

Project AIR FORCE is the FFRDC operated by RAND for the U.S. Air Force. It is the only Air Force FFRDC charged with policy analysis. Its chief mission is to conduct objective and independent research and analysis on enduring issues of policy, management, technology,

and resource allocation that will be of concern to the senior leaders and decisionmakers of the Air Force. Project AIR FORCE work is performed under contract F49620-91-C-0003. The research reported in this document was conducted under the C4I/Space Project within the Force Modernization and Employment Program of Project AIR FORCE.

The Arroyo Center is a studies and analysis FFRDC operated by RAND for the U.S. Army. It provides the Army with objective, independent analytic research on major policy and organizational concerns, emphasizing mid- and long-term problems. Arroyo Center work is performed under contract MDA903-91-C-0006. The research reported in this document was conducted under the C3I/Space for Contingency Operations Project within the Force Development and Technology Program of the Arroyo Center.

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This report is the second in a series devoted to presenting the analytical procedures and mathematical formulations required to construct a computer model of a military communication satellite system, load it efficiently with the radio signals required to support an operational scenario, and assess its vulnerability to jamming. It does not address the implementation of the model.

Like MR-639-AF/A, the first report of this series, the analysis is restricted to nonprocessing, frequency-translating (or "bent-pipe") transponders. However, instead of being operated in the linear mode, as was necessary there to accommodate the frequency-division-multiplexed, nonantijam signals being considered, the transponders here are considered to be driven deliberately into saturation and to behave like hard limiters. This is done to obtain the best possible performance (in a jamming environment) of the direct-sequence, spread-spectrum signals of interest. Such so-called antijam signaling is important to military communications because of the significant protection it can provide against jamming.

SYMBOLS

A	Receiving antenna collecting aperture, m^2
d	Subscript denoting downlink
E_b	Received bit energy, J
$(E_b/N_0)_{min}$	Bit detection threshold
E_d	Edge-of-coverage downlink EIRP, W
EIRP	Effective Isotropic Radiated Power, W
$E_{min,d}$	Minimum, downlink, edge-of-coverage EIRP required to achieve detection threshold
$E_{sat,d}$	Saturated, downlink, edge-of-coverage EIRP available from a given transponder-downlink antenna combination
G	Receiver antenna gain
H	Superscript denoting a hard limiter
H_d	Edge-of-coverage downlink power flux density at receiver, W/m^2
i	Subscript denoting input, presubscript denoting ith user
I	Interference power, W
J	Jammer power, W

J	Superscript referring to jammed case
k	Boltzmann's constant, 1.3806×10^{-23} J/K
K	Uplink jamming parameter
L	Superscript denoting a linear amplifier
n	Number of signals being considered
N_u	Satellite receiver effective noise power, W
N_0	Total interference power noise spectral density, W/Hz
o	Subscript denoting output
r	Slant range, m
R_b	Information bit rate, bps
S	Power of a given signal at the output of the transponder, W
T	Effective receiver noise temperature, K
u	Subscript denoting uplink
U	Superscript referring to unjammed case
W	Spread bandwidth, Hz
λ	Ratio of the powers of the sinusoidal and Gaussian (or steady and noise-like) components of the interference; also, wavelength, m
Λ	Small-signal suppression factor

Chapter One

INTRODUCTION

The basis for a concept-level modeling tool for military satellite communications was developed in a companion report, MR-639-AF/A, to which the reader is referred for details. Briefly, the modeling tool described therein permits evaluation of conceptual military communication satellite systems at a systems level. That is, it is restricted to consideration of only basic design parameters such as antenna gains; transponder gains, limiting characteristics, and power output; system noise temperatures; etc. Also, the initial analysis considers only nonantijam signals in nonprocessing satellites, which employ frequency-translating, wideband transponders, often referred to as "bent-pipe" transponders.

The analysis in MR-639-AF/A covers three major areas of research. The first concerns the systems aspects that relate to the use of such a modeling tool. Included are discussions of scenarios, earth stations, communication satellites, system configurations, analytical assumptions, and technical databases. The second area of research consists of the development of link formulas designed to permit "loading" the satellite; that is, developing the communication parameters required to accommodate users efficiently. The third area of research treats the effects of deliberate interference, i.e., jamming, on such unprotected systems.

The analysis presented here is concerned with the use of direct-sequence, spread-spectrum signals in nonprocessing satellites. Such signals are one of two major types of spread-spectrum signals designed to provide protection against jamming, the other being frequency-hopping, which will be treated in a follow-on analysis.

Basically, both types mitigate the effects of jamming by spreading the energy of the desired signal over a much wider bandwidth (the entire transponder bandwidth, in this case) than is required when using conventional nonantijam signaling techniques. When this spreading is done using a suitable pseudorandom sequence, it is possible to recover the desired information content while rejecting much of the interference from jammers and other friendly signals that occupy the same signaling band. A measure of the ability to do so is the so-called "processing gain," which is the ratio of the direct-sequence chipping rate (spread bandwidth) to the information data rate. The analysis presented here assumes that the reader is familiar with the properties of spread-spectrum signals. For the purposes of background, the direct-sequence type of spread-spectrum signaling considered here is one in which the signal energy is spread more or less uniformly across the entire spread band at all times.

The military application to which this analysis is most germane is one in which many low-duty-cycle users with similar equipment seek to communicate with one another using a single, hard-limiting, bandpass transponder in a geostationary communication satellite. To enhance security against intercept and jamming, it is assumed that the spreading codes have very long periods. To simplify operation, transmissions are assumed to be asynchronous at the chip level.

More detailed descriptions of spread-spectrum systems can be found in standard tests such as Dixon (1976), Holmes (1982), Simon, Omura, Scholtz, and Levitt (1985), and Nicholson (1988). The performance of such systems when used for multiple access communications is treated by Aein and Pickholtz (1982), Pursley, Sarwate, and Stark (1982), Geraniotis and Pursley (1982), and Baer (1982), which appear in a special issue of the *IEEE Transactions on Communications* devoted to spread-spectrum communications. Also contained in that issue is an excellent tutorial on spread-spectrum communications by Pickholtz, Schilling, and Milstein (1982).

An important difference from the analysis in the companion report is in the operating mode used for the transponder. When used to relay two or more nonantijam signals, a transponder is invariably designed to operate in its linear range. This is done to protect the nonantijam signals against the intermodulation products, to which they are particularly sensitive, that would otherwise be generated. As described

in the companion report, this makes such signals extremely vulnerable to jamming signals that can drive the transponder into the highly nonlinear saturation region, thereby not only suppressing the desired signals by power-sharing and nonlinear effects but also generating strong intermodulation products that would be harmful even if the signal suppression could be tolerated.

Because spread-spectrum signals are designed to operate in a hostile environment and can tolerate multipath, interference, and jamming, they are designed to have an inherent protection against interference. Note that nonorthogonal spread signals interfere with one another simply because they occupy the same spectral band. Thus, there is little incentive to operate the transponder in its linear region just to prevent the generation of intermodulation products, because the basic mutual interference is inherent in the very presence of the other direct-sequence, spread-spectrum signals.

Actually, there is little choice in the matter because a jammer, whose received power at the satellite greatly exceeds that of the desired signals, can drive the transponder well beyond saturation anyway.[1] Consequently, it is customary to provide nonprocessing satellites intended for use with spread-spectrum signals the ability to operate in a mode in which a hard limiter is followed by sufficient transponder gain to insure that the final amplifier is operated at its saturation level. Hence, the analysis presented here is largely concerned with the behavior of multiple direct-sequence, spread-spectrum signals and jamming that are passed through a hard limiter.

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[1]Such brute-force jamming is discussed in Chapter Nine of MR-639-AF/A, where it is shown that it can be accomplished by a jammer that, though small in comparison with a fixed jammer that approaches the state of the art, is not easily transportable or proliferated in large numbers.

Chapter Two

TRANSPONDER LOADING

The process for loading a transponder with nonantijam signals, as described in the companion report, is straightforward, because such signals are arranged in frequency-division multiplex and therefore are said to be orthogonal to one another (i.e., noninterfering). As a result, each signal needs only compete with the receiver noise at the earth terminal. This allows a loading process in which the signals are added one at a time until the transponder runs out of either power or bandwidth.

When direct-sequence, spread-spectrum signals are used, the process is complicated by the fact that each signal must contend not only with the receiver noise but also with all of the other signals and the intermodulation products produced by the hard-limiting action. Thus, although it is still possible to load a transponder by considering the signals one at a time and making adjustments to the previously loaded signals as new ones are added, it is simpler to try loading all of the signals simultaneously and making adjustments where necessary.

The equations needed to implement this procedure are derived in the appendix. It is shown there that if the transponder can be modeled as a bandpass hard limiter, and if the input to the transponder consists of a set of n constant-amplitude, direct-sequence, spread-spectrum signals for which it is always true that the sum of any $n - 1$ of these signals resembles Gaussian noise and has a total power much larger than the remaining signal, then the total power at the output is given, from Eq. (A.18), by

$$\underset{\text{output}}{\text{Total}} = \underset{\text{Desired signal}}{S} + \underbrace{\left[\left(\Lambda^U - 1\right)S + \Lambda^U \sum^{n-1} S\right]}_{\text{Total interference}} \tag{2.1}$$

where S denotes the power contained in any desired output signal and $\sum^{n-1} S$ is the power contained in the n – 1 other output signals. The quantity contained in brackets constitutes the total power in the output interference: The first term $(\Lambda^U - 1)S$ represents the intermodulation products produced by the desired signal; the second term $\Lambda^U \sum^{n-1} S$ represents the n – 1 other output signals plus the intermodulation products they produce. The superscript U refers to the unjammed case. Inasmuch as the interference is purely noise-like, the parameter γ, which is the ratio at the input of the steady to the noise-like component of the interference in the absence of jamming, is equal to zero. Then, from Eq. (A.2) or Figure A.1, it is seen that $\Lambda^U = 4/\pi$.

It is implicit in the foregoing that the thermal noise at the hard-limiter input can be neglected. Inasmuch as hard-limiting satellite transponders are usually designed to achieve hard limiting on input noise alone, this condition can be achieved only by insuring that the total uplink signal power at the satellite receiver input is much larger than the input thermal noise. Let ${}_iE_u$ denote the uplink EIRP of the ith user, i = 1, 2, · · · , n, where the subscript u will be used to identify uplink parameters. The uplink power flux density ${}_iH_u$ at the satellite is then given by

$${}_iH_u = \frac{{}_iE_u}{4\pi\left({}_ir_u\right)^2} \tag{2.2}$$

where ${}_ir_u$ is the slant range to the satellite. Let A_u denote the effective collecting aperture of the satellite receiving antenna. Then, the received uplink power ${}_iS_u$ from the ith user is given by

$${}_iS_u = {}_iH_u\, A_u = \left[\frac{{}_iE_u}{4\pi\left({}_ir_u\right)^2}\right]\frac{\lambda_u{}^2 G_u}{4\pi} = \left(\frac{\lambda_u}{4\pi\; {}_ir_u}\right)^2 {}_iE_u G_u \tag{2.3}$$

where A_u is related to the satellite receiver antenna gain G_u by

$$G_u = \frac{4\pi A_u}{\lambda_u^2} \tag{2.4}$$

and λ_u is the wavelength. For a direct-sequence, spread-spectrum system in which the users are all spread over a transponder having a bandwidth W, λ_u corresponds to the center frequency of the uplink channel of the transponder being used; λ_u will be approximately the same for all users.

To obtain the condition in which uplink noise can be neglected, let T_u denote the system noise temperature of the satellite receiver. The uplink thermal noise power N_u is then given by

$$N_u = kT_uW \tag{2.5}$$

where $k = 1.3806 \times 10^{-23}$ J/K is Boltzmann's constant. It will be assumed that the input noise can be neglected if

$$\sum_{i=1}^{n} {}_iS_u \gg N_u \tag{2.6}$$

or, using Eq. (2.3) to state the condition in terms of the uplink EIRPs,

$$\sum_{i=1}^{n} {}_iE_u\left(\frac{\lambda_u}{4\pi_i r_u}\right)^2 \gg \frac{kW}{(G/T)_u} \tag{2.7}$$

To satisfy the conditions leading to Eq. (2.1), it is further necessary that

$$\sum_{\substack{i=1 \\ i\neq j}}^{n} {}_iS_u \gg {}_jS_u \quad \text{all } j \quad n \gg 1 \tag{2.8}$$

or in terms of the uplink EIRPs, that

$$\sum_{\substack{i=1\\ i\neq j}}^{n} {}_iE_u\left(\frac{\lambda_u}{4\pi\,{}_ir_u}\right)^2 \gg {}_jE_u\left(\frac{\lambda_u}{4\pi\,{}_jr_u}\right)^2 \quad \text{all } j \quad n \gg 1 \tag{2.9}$$

Turning to the downlink, it may be noted that although the total power represented by Eq. (2.1) is stated in terms of the output power from the satellite, it is equally true when stated in terms of the received power at a given earth terminal provided the total interference is adjusted to include the receiver noise. To accommodate this, let T_d denote the effective noise temperature of the downlink receiver in degrees Kelvin, where the subscript d (here, as elsewhere) refers to the downlink. The total interference power noise spectral density N_0 at the receiver can then be written

$$N_0 = kT_d + \frac{1}{W}\left[\left(\Lambda^U - 1\right)S_d + \Lambda^U \sum^{n-1} S_d\right] \tag{2.10}$$

where k is Boltzmann's constant and W is the transponder bandwidth. To be detectable, the received power of the desired downlink signal to the ith user ${}_iS_d$ must satisfy the inequality

$$\begin{aligned} {}_iS_d &\geq N_0\left(\frac{{}_iE_b}{N_0}\right)_{min} {}_iR_b \\ &= \left\{kT_d + \frac{1}{W}\left[\left(\Lambda^U - 1\right)S_d + \Lambda^U \sum^{n-1} S_d\right]\right\}\left(\frac{{}_iE_b}{N_0}\right)_{min} {}_iR_b \end{aligned} \tag{2.11}$$

for all users. In Eq. (2.10) $({}_iE_b/\,{}_iN_0)_{min}$ denotes the bit detection threshold for the ith user as determined by the modulation and coding used, and ${}_iR_b$ denotes its information bit rate.[1]

[1]Suitable values of $(E_b/N_0)_{min}$ for coded system at a 10^{-6} bit error probability are presented in Table 3 of the companion report for various code rates.

To state Eq. (2.11) in terms of the downlink EIRPs, i.e., the ${}_iE_d$, note that Eq. (2.3) can be written

$${}_iS_d = \left(\frac{\lambda_d}{4\pi\,{}_ir_d}\right)^2 {}_iE_d\,{}_iG_d \tag{2.12}$$

where ${}_i\lambda_d$, ${}_ir_d$, and ${}_iG_d$ refer, respectively, to the downlink band-center wavelength, the downlink slant range, and the terrestrial receiver antenna gain for the ith user. Then, Eq. (2.11) becomes

$$\left(\frac{\lambda_d}{4\pi\,{}_ir_d}\right)^2 {}_iE_d\,{}_ir_d \geq$$
$$\left\{kT_d + \frac{1}{W}\left[\left(\Lambda^u - 1\right)_i E_d + \Lambda^u \sum^{n-1} E_d\right]\left(\frac{\lambda_d}{4\pi\,{}_ir_d}\right)^2 {}_iG_d\right\}\left(\frac{{}_iE_b}{N_0}\right)_{min} {}_iR_b \tag{2.13}$$

Applying Eq. (2.1) to the downlink, it is seen that

$$E_{sat,d} = {}_iE_d + \left[\left(\Lambda^u - 1\right)_i E_d + \Lambda^u \sum^{n-1} E_d\right] \tag{2.14}$$

where $E_{sat,d}$ is the saturated, edge-of-coverage, downlink EIRP for the transponder-downlink antenna combination being considered. Eliminating the quantity in brackets common to Eqs. (2.13) and (2.14) then leads to the minimum edge-of-coverage downlink EIRP ${}_iE_{min,d}$ required to achieve threshold

$${}_iE_{min,d} = \frac{\dfrac{kW}{{}_i(G/T)_d}\left(\dfrac{4\pi\,{}_ir_d}{\lambda_d}\right) + E_{sat,d}}{\dfrac{W/{}_iR_b}{{}_i(E_b/N_0)_{min}} + 1} \tag{2.15}$$

which must be satisfied for each downlink. The concomitant condition on total signal EIRP is obtained from Eq. (2.14) by noting that

each downlink signal of EIRP ${}_iE_d$ is accompanied by intermodulation products of EIRP $(\Lambda^u - 1)_i E_d$. Thus, when summed, the result is

$$E_{sat,d} = \sum_{i=1}^{n} \left[{}_iE_d + \left(\Lambda^u - 1\right) {}_iE_d \right]$$

or

$$\sum_{i=1}^{n} {}_iE_d = \frac{1}{\Lambda^u} E_{sat,d} \tag{2.16}$$

In terms of ${}_iE_{min,d}$ from Eq. (2.15), this requires that

$$\sum_{i=1}^{n} {}_iE_{min,d} \le \frac{1}{\Lambda^u} E_{sat,d} \tag{2.17}$$

The transponder is used as efficiently as possible when it is loaded such that equality is attained in Eq. (2.17). However, inasmuch as every signal will then be just at threshold, the addition of even one more signal (or, in particular, the addition of a jammer) will cause every signal to fall below threshold. It is clear that a secure (against jamming) system controller is required to prevent self-induced chaos in the absence of jamming and to regulate usage in its presence.

If the most efficient transponder use is desired, signals can be added until the solutions to Eq. (2.15) come as close to equality as possible in Eq. (2.17). The system will then have virtually no jam resistance, but that may be considered satisfactory when jamming is considered unlikely. If jamming appears, the system controller will then need to reduce data rates or remove users until operation becomes satisfactory. (The effects of jamming and the extent of the measures necessary to mitigate them are considered in Chapter Three. The subject of jamming is introduced at this point to show, in general terms, how the anticipation of jamming affects the loading philosophy.)

An alternative approach is to deliberately load the satellite lightly so that some level of jamming can be tolerated without the need for intervention by a system controller. If, for example, a level of protec-

tion LP dB is desired, the loading using Eq. (2.15) can be stopped when

$$\sum_{i=1}^{n} {}_iE_{min,d} = \frac{10^{-(LP)/10}}{\Lambda^u} E_{sat,d} \tag{2.18}$$

The actual downlink edge-of-coverage radiated EIRPs ${}_iE_d$ assigned to the various users would then be given by

$${}_iE_d = 10^{LP/10} {}_iE_{min,d} \tag{2.19}$$

The transponder loading procedure using these equations is as follows:

1. Select a transponder of bandwidth W, input sensitivity $(G/T)_u$, uplink band-center wavelength λ_u, downlink band-center wavelength λ_d, and downlink, edge-of-coverage, saturated output EIRP $E_{sat,d}$.
2. Select a set of n uplink users $n >> 1$ for which the ith user has a maximum uplink EIRP ${}_iE_{sat,u}$, is at a slant range ${}_ir_u$ from the satellite, and requires a data rate ${}_ir_b$.
3. Select a corresponding set of n downlink users for which the ith user is at a slant range ${}_ir_d$ and has a receiver of sensitivity ${}_i(G/T)_d$ and a detection threshold ${}_i(E_b/N_0)_{min}$ for the desired bit error probability and code rate.
4. Calculate the minimum downlink EIRP ${}_iE_{min,d}$ required for each user from Eq. (2.15).
5. Verify that Eq. (2.17) or Eq. (2.8) is satisfied. If they are not satisfied, signals must be removed until they are. If they are satisfied, signals may be added until equality is approached as closely as possible.
6. The uplink EIRPs ${}_iE_u$ must be made proportional to the downlink EIRPs ${}_iE_{min,d}$ (or ${}_iE_d$) with the constant of proportionality chosen such that Eqs. (2.7) and (2.9) are satisfied and that no ${}_iE_u$ exceeds its limiting value ${}_iE_{sat,u}$. If any one of these conditions is not satisfied, users must be removed until all are satisfied.

Chapter Three

THE EFFECT OF JAMMING

The effect of introducing deliberate interference, i.e., jamming, into a hard-limiting bandpass transponder loaded with direct-sequence, spread-spectrum signals is analyzed in the appendix. The principal result is given by Eq. (A.27), which, following the notation introduced in Chapter Two, can be written

$$\begin{array}{c}\text{Total}\\\text{output}\end{array} = \underbrace{\frac{\Lambda^U}{\Lambda^J} KS}_{\text{Desired signal}} + \underbrace{\left[\left(\Lambda^U - \frac{\Lambda^U}{\Lambda^J} K\right) S + \Lambda^U \sum^{n-1} S\right]}_{\text{Total interference}} \tag{3.1}$$

where the superscript J denotes the jammed case and where the uplink jamming parameter K is given by Eq. (A.21) as

$$K = \frac{1}{1 + (J/S)_u} \tag{3.2}$$

where the subscript u, to denote the uplink input, has been substituted for i, which was used in the appendix. Depending on the nature and level of the jamming, the parameter γ^J, which is the ratio of the steady to the noise-like components of the resulting total interference, can have virtually any nonnegative value. From Eq. (A.2) and Figure A.1, it is then seen that

$$\frac{4}{\pi} \leq \Lambda^J < 4 \tag{3.3}$$

where the value $4/\pi$ obtains when J is noise-like or consists of the sum of many roughly equal-amplitude CW tones. When J is a pure sinusoid, even if noise modulated, Λ^J can approach 4 when J is large in comparison with $\overset{n-1}{\Sigma}\,S$, the sum of the other n – 1 signals. As noted in Chapter Two, $\Lambda^U = 4/\pi$.

If Eq. (3.1) is compared with Eq. (2.1), it is seen that the principal effect of jamming is to reduce the level of the desired signal by the factor $\Lambda^U K/\Lambda^J$, which can be substantial, and to increase the total interference slightly by decreasing the subtractive term from unity to $\Lambda^U K/\Lambda^J$, which will be much less than unity for $J/S \gg 1$. Quantitatively, the effect of jamming on the ith signal is best displayed by forming the ratio of the downlink signal-to-interference ratios in the jammed and unjammed cases. Thus, from Eq. (2.1),

$$\begin{aligned}\left({}_iS/I\right)^U &= \frac{{}_iS^U}{\left(\Lambda^U - 1\right)\,{}_iS^U + \Lambda^U\left(\overset{n-1}{\Sigma}\,S\right)^J} \\ &= \frac{1}{\Lambda^U\left\{\left(1 - 1/\Lambda^U\right) + \left[\left(\overset{n-1}{\Sigma}\,S\Big/{}_iS\right)\right]^U\right\}}\end{aligned} \tag{3.4}$$

and from Eq. (3.1),

$$\begin{aligned}\left({}_iS/I\right)^J &= \frac{\Lambda^U K\,{}_iS^J/\Lambda^J}{\left(\Lambda^U - \Lambda^U K/\Lambda^J\right)\,{}_iS^J + \Lambda^U\left(\overset{n-1}{\Sigma}\,S\right)^J} \\ &= \frac{K}{\Lambda^J\left\{\left(1 - K/\Lambda^J\right) + \left[\left(\overset{n-1}{\Sigma}\,S\Big/{}_iS\right)\right]^J\right\}}\end{aligned} \tag{3.5}$$

Then, the jamming factor $(JF)^i$ for the ith downlink signal is given by the ratio of these signal-to-interference ratios, or

$$_i(JF)\;\frac{\left({}_iS/I\right)_d^U}{\left({}_iS/I\right)_d^J}=\frac{\Lambda^J\left\{\left(1-K/\Lambda^J\right)+\left[\left(\overset{n-1}{\Sigma}\,S/{}_iS\right)\right]^J\right\}}{\Lambda^U K\left\{\left(1-1/\Lambda^J\right)+\left[\left(\overset{n-1}{\Sigma}\,S/{}_iS\right)\right]^U\right\}} \tag{3.6}$$

From the condition given by Eq. (A.3), it follows that the Σ terms dominate, yielding

$$_i(JF)\cong\frac{\Lambda^J}{\Lambda^U K}\,\frac{\left[\left(\overset{n-1}{\Sigma}\,S/{}_iS\right)\right]^J}{\left[\left(\overset{n-1}{\Sigma}\,S/{}_iS\right)\right]^U}=\frac{\Lambda^J}{\Lambda^U K} \tag{3.7}$$

It is recognized that jamming does not alter the ratio $\left(\overset{n-1}{\Sigma}/{}_iS\right)$. Then, applying Eq. (3.2) leads to

$$_i(JF)\;=\frac{\Lambda^J}{\Lambda^U}\left[1+{}_i(J/S)_u\right] \tag{3.8}$$

which is the desired result. The jamming factor for the ith downlink signal is seen to depend only on its uplink jam-to-signal ratio ${}_i(J/S)_u$ at the satellite input. Thus, the jamming factor can be calculated uniquely for each signal.

The utility of the jamming factor as a measure of the effectiveness of jamming derives from the fact that if the downlink earth-terminal receiver noise is small in comparison with interfering direct-sequence, spread-spectrum signals, as is usually the case, then the signal-to-interference ratio reduction for the ith signal, which is what the jamming factor calculates, equals the amount by which the level-

of-protection factor LP defined in Chapter Two is reduced by the jamming. If the jamming factor for a given signal exceeds the level-of-protection factor, that signal is driven below its detection threshold and is, therefore, disabled.

At the same time, the excess of the jamming factor in comparison with the level-of-protection factor also equals the amount by which the spreading must be increased to cope with the jamming. If, for example, LP = 20 dB and there is CW jamming with ${}_i(J/S)_u = 20$ dB for the ith signal, then, from Eq. (3.8), ${}_i(JF) = 4/(4/\pi)(1+10^2) = 317.3 \Rightarrow 25.0$ dB and the data rate for that signal must be reduced by a factor of at least 3.16 (i.e., 5 dB) if it is to remain usable.

Appendix

POWER SHARING AND SMALL-SIGNAL SUPPRESSION IN A HARD LIMITER

The power sharing and small-signal suppression analysis presented in Appendix B of MR-639-AF/A was concerned with the effect of jamming on a transponder loaded with nonantijam signals and operating in its linear range in the absence of jamming. The concern here is with a hard-limiting bandpass amplifier (or, simply, a hard limiter) loaded with direct-sequence, spread-spectrum signals in both the absence and presence of jamming. The purpose is to determine, in both cases, the output level of the desired signal and of the interference to which it is subjected. The analytical procedure is to equate the output of a hypothetical linear amplifier to that of a hard limiter first without jamming and then with jamming. This permits the introduction of the small-signal suppression factor Λ introduced by Cahn (1961), in terms of which the desired results are then stated.

Cahn's analysis shows that if the input to a bandpass hard limiter consists of a sinusoidal signal[1] of power S_i and an interference of power N_i consisting of the sum of a sinusoidal component and a Gaussian noise component, then the ratio of the input and output signal-to-interference power ratios is given by

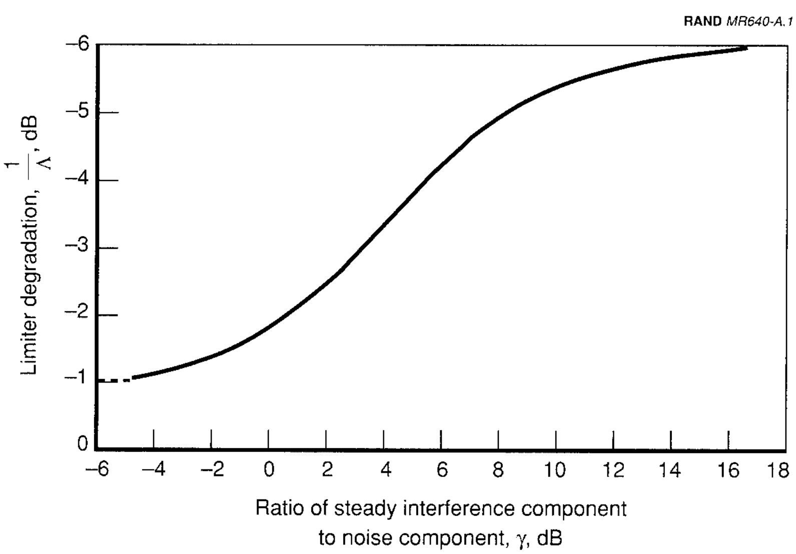
[1]Cahn's analysis permits the sinusoidal signal and interference components each to have any desired frequency within the hard-limiter passband. Cahn shows that the result is valid for constant-envelope signals in correlation systems, such as the one used here to demodulate direct-sequence, spread-spectrum signals.

$$\frac{(S/I)_i}{(S/I)_o} = \Lambda \tag{A.1}$$

where

$$\frac{1}{\Lambda} = \frac{\pi}{4}(\gamma + 1)\left[e^{-\gamma/2} I_0(\gamma/2)\right]^2 \tag{A.2}$$

in which I_0 is a modified Bessel function of the first kind of order zero, and γ is the ratio of the powers of the sinusoidal and Gaussian (or steady and noise-like) components of the interference. The quantity $1/\Lambda$, which is plotted in Figure A.1, is usually referred to as the limiter degradation or suppression factor. Cahn's analysis is valid when the output interference power is large compared with the output signal power. It can be seen from Figure A.1 that when the



SOURCE: C. R. Cahn, "A Note on Signal-to-Noise Ratio in Band-Pass Limiters," *Trans. IRE*, Vol. IT-7, No. 1, January (1961). © 1961 IRE.

Figure A.1—Small-Signal Suppression in a Hard-Limiting Bandpass Amplifier

interference is largely noise-like ($\gamma<<1$), the signal-to-interference ratio is degraded by no less than a factor of $4/\pi$, or 1.05 dB. When the interference is largely steady ($\gamma>>1$), the signal-to-interference ratio is degraded by a factor of as much as 4, or 6.02 dB.

To apply the foregoing, consider the linear amplifier and hard-limiter diagrams shown in Figure A.2a, when jamming is absent, and in Figure A.2b, when it is present. In both cases, the input includes a number of desired signals of roughly equal power. These signals are all phase modulated and have constant envelopes. Thus, when any one of them is considered as the signal of interest for the purposes of analysis, Cahn's requirement of a sinusoidal input signal is satisfied. To satisfy Cahn's other requirement, i.e., that the output interference power be large in comparison with the output signal power, assume that the other input signals (which constitute the interference power) are sufficient in number and power that their sum can be considered a Gaussian input interference that is large in comparison with the input signal of interest.[2] That is,

$$\Sigma_i >> S_i \tag{A.3}$$

where S_i denotes the power of the input signal of interest and Σ_i the power of the sum of the other input signals. In the absence of jamming, Σ_i constitutes the entire input interference power. Inasmuch as the signal of interest is suppressed by the hard limiter and inasmuch as the signal power lost thereby appears as intermodulation products in the output, thus adding to the output interference power, it follows that

$$I_o = \Sigma_o + (IM)_o >> S_o \tag{A.4}$$

The interference is denoted by I and the intermodulation products by (IM), with the subscript o denoting quantities at the output. Cahn's other requirement is thus satisfied.

[2]The direct-sequence, spread-spectrum signals considered here are secure with very long periods. The receiver has a relatively short integration time with the longest being typically a 10 second integration time by the code tracking loop. Over such a short integration time with respect to the code period, the signals are independent and uncorrelated.

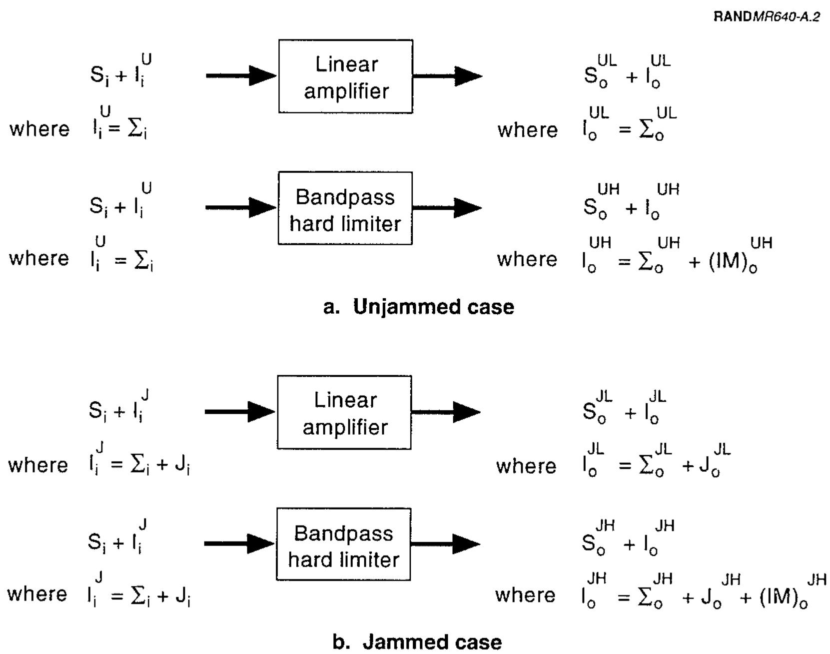


Figure A.2—Configuration of a Linear Amplifier and a Bandpass Hard Limiter in the Absence and Presence of Jamming

In the absence of jamming, the above assumptions yield $\gamma^U = 0$ so $\Lambda^U = 4/\pi$, where the superscript U signifies the unjammed case. In the presence of jamming,[3] denoted by the superscript J, the value of γ^J depends on the nature of the jamming and its power relative to Σ_i. If the jamming is noise-like, it simply adds to Σ_i and the total interference remains noise-like, thereby yielding $\gamma^J=0$ and

[3]Repeat jammers and multitone jammers are not of interest here because they are effective only against frequency-hopped, spread-spectrum signals. Constant-amplitude jammers are the most effective type against direct-sequence, spread-spectrum signals of the type considered here. However, they have the defect that if they are pure sinusoids or swept tones, they can be largely negated by adaptive spectral-notch filtering. Though such filtering can be very effective, the advantage to be gained cannot exceed that which will be achieved by forcing the jammer to abandon constant-envelope jamming in favor of Gaussian noise jamming (against which filtering is ineffective).

$\Lambda^J = \Lambda^U = 4/\pi$, regardless of the level of the jamming. If the jamming is sinusoidal, the value of γ^J depends on the ratio of the input jamming power J_i to the sum of the other input signals Σ_i. At low levels of jamming, the total input interference may appear largely noise-like, thereby yielding $\gamma^J<1$ and values of Λ^J not much greater than $4/\pi$. At high levels of jamming, the total interference may appear largely steady, thereby yielding $\gamma^J>1$ and values of Λ approaching 4.

As an aid to distinguishing the components of the input and output quantities clearly, both are always stated as the sum of the signal of interest and an interference, which is then given separately at each point. The input signal of interest S_i and the sum of the other input signals Σ_i are shown without superscripts because they are the same for all cases. In the unjammed case, the input interference I_i^U is simply equal to the sum of the other input signals Σ_i i.e.,

$$I_i^U = \Sigma_i \tag{A.5}$$

In the jammed case, the input interference also includes the jamming signal J_i so,

$$I_i^J = \Sigma_i + J_i \tag{A.6}$$

It is assumed that the input (i.e., uplink) noise power is negligible in comparison with Σ_i, the sum of the other input signals.

In the outputs, an additional superscript is required to distinguish between the outputs of the linear amplifier L and the hard limiter H. The output interferences I_o^{UL} and I_o^{JL} for the linear amplifier are straightforward and are simply

$$I_o^{UL} = \Sigma_o^{UL} \tag{A.7}$$

and

$$I_o^{JL} = \Sigma_o^{JL} + J_o^{JL} \tag{A.8}$$

where the second superscript has been added. In the case of the hard limiter, the output interferences I_o^{UH} and I_o^{JH} require the inclusion of an intermodulation term so they are written

$$I_o^{UH} = \Sigma_o^{UH} + (IM)_o^{UH} \tag{A.9}$$

and

$$I_o^{JH} = \Sigma_o^{JH} + J_o^{JH} + (IM)_o^{JH} \tag{A.10}$$

as indicated in Figures A.2a and A.2b. It should be emphasized that S_o^{UH}, Σ_o^{UH}, S_o^{JH}, and Σ_o^{JH} represent the useful (i.e., undistorted) components of the output signals. Because of the hard-limiting process, these terms contribute to the output intermodulation products by an amount equal to the small-signal suppression. Of course, the total interference includes not only these intermodulation products but also the undistorted other signals Σ_o^{UH} and Σ_o^{JH} as well as the jamming J_o^{JH}.

Cahn's result, Eq. (A.1), may now be applied to each configuration in Figure A.2 thereby yielding the input-output relationships

Unjammed linear amplifier
$$\frac{(S/I)_i^{UL}}{(S/I)_o^{UL}} = 1 \quad \text{or} \quad \frac{I_o^{UL}}{S_o^{UL}} = \frac{I_i^{U}}{S_i} \tag{A.11}$$

Unjammed hard limiter
$$\frac{(S/I)_i^{UH}}{(S/I)_o^{UH}} = \Lambda^U \quad \text{or} \quad \frac{I_o^{UH}}{S_o^{UH}} = \Lambda^U \frac{I_i^{U}}{S_i} \tag{A.12}$$

Jammed linear amplifier
$$\frac{(S/I)_i^{JL}}{(S/I)_o^{JL}} = 1 \quad \text{or} \quad \frac{I_o^{JL}}{S_o^{JL}} = \frac{I_i^{J}}{S_i} \tag{A.13}$$

Jammed hard limiter
$$\frac{(S/I)_i^{JH}}{(S/I)_o^{JH}} = \Lambda^J \quad \text{or} \quad \frac{I_o^{JH}}{S_o^{JH}} = \Lambda^J \frac{I_i^J}{S_i} \qquad \text{(A.14)}$$

As noted above, $\Lambda^U = 4/\pi$ and $4/\pi \le \Lambda^J < 4$.

A useful relationship between desired output signals from the linear amplifier and the hard limiter in the unjammed case can be obtained by equating their total outputs. Thus, from Figure A.2a,

$$S_o^{UL} + I_o^{UL} = S_o^{UH} + I_o^{UH}$$

$$1 + \frac{I_o^{UL}}{S_o^{UL}} = \frac{S_o^{UH}}{S_o^{UL}} + \frac{I_o^{UH}}{S_o^{UH}} \frac{S_o^{UH}}{S_o^{UL}}$$

$$1 + \frac{I_i^U}{S_i} = \frac{S_o^{UH}}{S_o^{UL}} \left(1 + \Lambda^U \frac{I_i^U}{S_i} \right)$$

where Eqs. (A.11) and (A.12) have been used. Rearranging leads to

$$\frac{S_o^{UH}}{S_o^{UL}} = \frac{S_i + I_i^U}{S_i + \Lambda^U I_i^U}$$

However, $I_i^U >> S_i$, so

$$\frac{S_o^{UH}}{S_o^{UL}} = \frac{1}{\Lambda^U} \qquad \text{(A.15)}$$

which is the desired relationship.

The breakdown of the output into the desired signal and the total interference with which it must compete is found by again equating the outputs of the linear amplifier and the hard limiter, but this time breaking out the output interferences in terms of their constituent components, as given by Eqs. (A.7) and (A.9). Thus, from Figure A.2a,

$$\begin{aligned} S_o^{UL} + \Sigma_o^{UL} &= S_o^{UH} + \Sigma_o^{UH} + (IM)_o^{UH} \\ &= \frac{1}{\Lambda^U} S_o^{UL} + \frac{1}{\Lambda^U} \Sigma_o^{UL} + (IM)_o^{UH} \end{aligned} \tag{A.16}$$

where Eq. (A.15) has been applied to all of the hard-limiter output signals. This leads to

$$\begin{aligned} (IM)_o^{UH} &= S_o^{UL}\left(1 - \frac{1}{\Lambda^U}\right) + \Sigma_o^{UL}\left(1 - \frac{1}{\Lambda^U}\right) \\ &= \left(\Lambda^U - 1\right) S_o^{UH} + \left(\Lambda^U - 1\right) \Sigma_o^{UH} \end{aligned} \tag{A.17}$$

where Eq. (A.15) has again been applied. Substituting Eq. (A.17) into Eq. (A.16) then yields

$$\underset{\text{output}}{\text{Total}} = \underbrace{S_o^{UH}}_{\substack{\text{Desired} \\ \text{signal}}} + \underbrace{\left[\left(\Lambda^U - 1\right) S_o^{UH} + \Lambda^U \Sigma_o^{UH}\right]}_{\substack{\text{Total} \\ \text{interference}}} \quad \begin{matrix}\text{Unjammed} \\ \text{hard limiter}\end{matrix} \tag{A.18}$$

where the term in S_o^{UH} in Eq. (A.18) has not been recombined with S_o^{UH} in Eq. (A.17) because the quantity $(\Lambda^U - 1)S_o^{UH}$ has been identified in Eq. (A.18) as that component of the output intermodulation products contributed by the desired signal. Equation (A.18) is the desired final result for the unjammed case and will be used to load the satellite.

The jammed case is treated in a similar fashion by equating the outputs of the linear amplifier and the hard limiter when jammed. Thus, from Figure A.2b,

$$
\begin{aligned}
S_o^{JL} + I_o^{JL} &= S_o^{JH} + I_o^{JH} \\
1 + \frac{I_o^{JL}}{S_o^{JL}} &= \frac{S_o^{JH}}{S_o^{JL}} + \frac{I_o^{JH}}{S_o^{JH}}\frac{S_o^{JH}}{S_o^{JL}} \\
1 + \frac{I_i^{J}}{S_i} &= \frac{S_o^{JH}}{S_o^{JL}}\left(1 + \Lambda^J \frac{I_i^J}{S_i}\right)
\end{aligned}
$$

where Eqs. (A.13) and (A.14) have been used. Rearranging leads to

$$
\frac{S_o^{JH}}{S_o^{JL}} = \frac{S_i + I_i^J}{S_i + \Lambda^J I_i^U}
$$

However, $I_i^J >> S_i$, so

$$
\frac{S_o^{JH}}{S_o^{JL}} = \frac{1}{\Lambda^J} \tag{A.19}
$$

which is the counterpart to Eq. (A.15) when there is jamming. A relationship between the jammed linear amplifier and the unjammed hard limiter is also desired. Equating their outputs from Figure A.2 yields

$$
\begin{aligned}
S_o^{JL} + I_o^{JL} &= S_o^{UH} + I_o^{UH} \\
1 + \frac{I_o^{JL}}{S_o^{JL}} &= \frac{S_o^{UH}}{S_o^{JL}} + \frac{I_o^{UH}}{S_o^{UH}}\frac{S_o^{UH}}{S_o^{JL}} \\
1 + \frac{I_i^{J}}{S_i} &= \frac{S_o^{UH}}{S_o^{JL}}\left(1 + \Lambda^U \frac{I_i^U}{S_i}\right)
\end{aligned}
$$

where Eqs. (A.12) and (A.13) have been applied. Rearranging leads to

$$
\frac{S_o^{UH}}{S_o^{JL}} = \frac{S_i + I_i^J}{S_i + \Lambda^U I_i^U}
$$

However, $I_i^J >> S_i$ and $I_i^U >> S_i$, so

$$\frac{S_o^{UH}}{S_o^{JL}} = \frac{I_i^J}{\Lambda^U I_i^U} = \frac{1}{\Lambda^U}\frac{S_i + J_i}{S_i} = \frac{1}{K\Lambda^U} \tag{A.20}$$

where

$$K = \frac{S_i}{S_i + J_i} = \frac{1}{1 + (J/S)_i} \tag{A.21}$$

The decomposition of the output into the desired signal and the total interference with which it must compete is found by again equating the outputs of the jammed linear amplifier and the jammed hard limiter. Thus, from Figure A.2b

$$\begin{aligned} S_o^{JL} + \Sigma_o^{JL} + J_o^{JL} &= S_o^{JH} + \Sigma_o^{JH} + J_o^{JH} + (IM)_o^{JH} \\ &= \frac{1}{\Lambda^S} S_o^{JL} + \frac{1}{\Lambda^J}\Sigma_o^{JL} + J_o^{JH} + (IM)_o^{JH} \end{aligned} \tag{A.22}$$

where Eq. (A.19) has been applied to all of the hard-limiter output signals. This leads to

$$(IM)_o^{JH} = \left(1 - \frac{1}{\Lambda^J}\right) S_o^{JL} + \left(1 - \frac{1}{\Lambda^J}\right)\Sigma_o^{JL} + \left(J_o^{JL} - J_o^{JH}\right)$$

which, when substituted back into Eq. (A.22), yields

$$\begin{matrix}\text{Total} \\ \text{output}\end{matrix} = \underbrace{S_o^{JH}}_{\text{Desired signal}} + \underbrace{\left[\left(1 - \frac{1}{\Lambda^J}\right) S_o^{JL} + \Sigma_o^{JL} + J_o^{JL}\right]}_{\text{Total interference}} \tag{A.23}$$

Substituting from Eqs. (A.19) and (A.20) then leads to

$$\begin{aligned}\begin{matrix}\text{Total}\\\text{output}\end{matrix} &= \underbrace{\frac{\Lambda^U}{\Lambda^J} K S_o^{UH}}_{\substack{\text{Desired}\\\text{signal}}} \\ &+ \underbrace{\left[\left(1-\frac{1}{\Lambda^J}\right)\Lambda^U K S_o^{UH} + \Lambda^U K\,\Sigma_o^{UH} + J_o^{JL}\right]}_{\substack{\text{Total}\\\text{interference}}}\end{aligned} \tag{A.24}$$

It remains to evaluate J_o^{JL}. To do this, equate Eq. (A.24) to Eq. (A.18) to get

$$\begin{aligned}S_o^{UH} + \left[\left(\Lambda^U - 1\right)S_o^{UH} + \Lambda^U \Sigma_o^{UH}\right] = \\ \frac{\Lambda^U}{\Lambda^J} K S_o^{UH} + \left[\left(1-\frac{1}{\Lambda^J}\right)\Lambda^U K S_o^{UH} + \Lambda^U K \Sigma_o^{UH} + J_o^{JL}\right]\end{aligned}$$

$$\Lambda^U S_o^{UH} + \Lambda^U \Sigma_o^{UH} = \Lambda^U K S_o^{UH} + \Lambda^U K \Sigma_o^{UH} + J_o^{JL}$$

which yields

$$J_o^{JL} = \Lambda^U (1-K)\left(S_o^{UH} + \Sigma_o^{UH}\right) \tag{A.25}$$

Substituting Eq. (A.25) into Eq. (A.24) then yields a total interference given by

$$\begin{aligned}
\begin{matrix}\text{Total}\\ \text{int erference}\end{matrix} =& \\
&\left(1-\frac{1}{\Lambda^J}\right)\Lambda^U K S_o^{UH} + \Lambda^U K\, \Sigma_o^{UH} + \Lambda^U (1-K)\left(S_o^{UH} + \Sigma_o^{UH}\right) \\
&= \Lambda^U K\left(S_o^{UH} + \Sigma_o^{UH}\right) - \frac{\Lambda^U}{\Lambda^J} K S_o^{UH} + \Lambda^U (1-K)\left(S_o^{UH} + \Sigma_o^{UH}\right) \\
&= \Lambda^U\left(S_o^{UH} + \Sigma_o^{UH}\right) - \frac{\Lambda^U}{\Lambda^J} K S_o^{UH} \\
&= \Lambda^U\left[\left(1-\frac{K}{\Lambda^J}\right) S_o^{UH} + \Sigma_o^{UH}\right]
\end{aligned} \tag{A.26}$$

Finally, substituting back into Eq. (A.21) yields

$$\begin{matrix}\text{Total}\\ \text{output}\end{matrix} = \underbrace{\frac{\Lambda^U}{\Lambda^J} K S_o^{UH}}_{\begin{matrix}\text{Desired}\\ \text{signal}\end{matrix}} + \underbrace{\Lambda^U\left[\left(1-\frac{K}{\Lambda^J}\right) S_o^{UH} + \Sigma_o^{UH}\right]}_{\begin{matrix}\text{Total}\\ \text{int erference}\end{matrix}} \quad \begin{matrix}\text{Jammed}\\ \text{hard}\\ \text{lim iter}\end{matrix} \tag{A.27}$$

which is the desired result. Note that the desired signal S_o^{JH} is given by the leading term

$$S_o^{JH} = \frac{\Lambda^U}{\Lambda^J} K S_o^{UH} \tag{A.28}$$

which demonstrates that the effect of the jamming is to suppress the unjammed hard-limiter output S_o^{JH} by the factor $(\Lambda^U / \Lambda^J)K$, where K is given by Eq. (A.21). The factor (Λ^U / Λ^J) gives the additional small-signal suppression caused by the jammer, whereas the factor K gives the power-sharing reduction. Equation (A.27) is the desired final result for the jammed case and will be used to assess the effect of jamming.

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